

ГЕОЛОШКИ АНАЛИ БАЛКАНСКОГА ПОЛУОСТРВА ANNALES GÉOLOGIQUES DE LA PÉNINSULE BALKANIQUE	65 (2002–2003)	101–105	БЕОГРАД, ДЕЦЕМБАР 2004 BELGRADE, December 2004
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The activation energy of viscoelastic deformation of rocks

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Abstract. The suggested model, which considers the simultaneous appearance (on the intermolecular scale) of elastic and viscous behaviours of rocks during their viscoelastic deformation, enables the relaxation time, t_0 , of the viscoelastic deformation energy to be defined. By determination of t_0 , the activation energies of the elastic and viscous deformation can be separated from the temperature dependence of the viscoelastic deformation energy stream U . The probabilistic approach to viscoelastic deformation of rocks allows the introduction of the a notion of the total probability, consisting of the probabilities of elastic and viscous deformations.

Keywords: viscoelastic, deformation, activation energy, energy stream, relaxation time.

Абстракт. Модел који је предложен у овом раду приказује симултану појаву (у интермолекуларној размери) еластичног и вискозног понашања стена током њиховог вискоеластичног деформисања. На тај начин омогућено је дефинисање времена релаксације t_0 вискоеластичне деформационе енергије. Када се ово време одреди онда могу да се раздвоје активирани енергије еластичних и вискозних деформација од температурне зависности вискоеластичног деформационог енергетског тока U . Пробабилистички приступ, примењен на вискоеластичне деформације стена, омогућава да се уведе појам тоталне вероватноће која се састоји од вероватноћа еластичних и вискозних деформација.

Кључне речи: вискоеластичност, деформација, активирана енергија, енергетски ток, време релаксације.

Introduction

At present, there is great interest in the viscoelastic deformation of rocks, since the viscoelastic bodies are heterogeneous and complex, in particular the deformation of rocks which elapse at extraordinarily various physical conditions existing in the interior of the earth (e.g. TROITZKAYA, 1961).

The crust rocks at different natural conditions are both elastic and brittle solids. The ability of crust and the mantle rocks to transmit seismic waves, when the forces act during very short times, testifies to their high elasticity of compression and displacement (e.g. VERHOOGEN *et al.*, 1974, TURCOTT & SCHUBERT, 1985). However, there is evidence for the elastic behaviour of rocks at more long stresses, for example Chandler oscillations of the pole and earth tide. On the other hand, the brittle behaviour of rocks in the crust upper is also verified by the existence of numerous natural break surfaces on all scales (breaks, cracks, etc.), (e.g. VERHOOGEN *et al.*, 1974, TURCOTT & SCHUBERT, 1985). There are other data (e.g. VERHOOGEN *et al.*, 1974), which

confirm: the viscous flow of rocks in the solid state without large breaks; the frequent appearance of inelastic and residual natural deformations in rocks. The tendency of the crust to a hydrostatic equilibrium allows the assumption that some uppers of the earth's crust behave like a very viscous liquid in their properties. Crust rock flow in the solid state on a small scale is proven by such structures as foliation, orogeny and the deformation of bodies with certain primary sizes or shapes.

The above stated data point to the fact that elastic and viscous deformations occur simultaneously in the crust.

In the theory of macroscopic elasticity of viscoelastic deformation, the value of the characteristic time is usually given, during which the elastic behaviour changes into viscous behaviour. This is a rough model of rocks: elastic under the influence of momentary forces and viscous under the influence of the long duration of tectonic forces. In reality both behaviours (elastic and viscous) appear simultaneously (e.g. ZELINSKY & MELKONYAN, 1997). TROITZKAYA (1961) and VERHOOGEN *et al.* (1974), assumed that the elastic behaviour of a body is

slowed down by the viscous component. For example, assume that during a given time (let it be assumed, ~ 100 years), the elastic behaviour of a substance completely changes into viscous. However the viscous behaviour of a deformable body appears during the deformation process (on the intermolecular scale), (e.g. LANDAU & LIFSHITS, 1987), though the body is elastic as a whole. Such an assumption allows the fact that the intermolecular forces have an insignificant radius of action (e.g. LANDAU & LIFSHITS, 1987). In spite of the fact that the influence of the forces extends around, thus making them particles only at an intermolecular distance, it simultaneously creates a residual phenomena, which can be involved even at the beginning of the deformation processes.

The energy stream accumulation and relaxation at viscoelastic deformation

The purpose of this work was to investigate the viscoelastic deformations of rocks and to determine their activation energies taking into account the simultaneous appearance of elastic and viscous behaviours and deformation energies of rocks. The viscoelastic deformation is considered in certain deformable plane of rocks with thickness l .

For a deformation ε , (e.g. VERHOOGEN *et al.*, 1974), the following equation can be written:

$$\frac{\partial \varepsilon(x,t)}{\partial t} = \frac{1}{\eta} \frac{\partial \tau(x,t)}{\partial x} - \frac{\varepsilon(x,t)}{t_0} \quad (1)$$

where η is the viscosity, τ is the stress, $t_0 = \eta/\mu$ is the relaxation time of viscoelastic deformation, where μ is the shear modulus. From the time t_0 , the relaxation time of the energy of viscoelastic deformation can be identified. The axis x is normal to the deformable plane.

Eqn. (1) is valid for linear deformation, i.e. $t_0 = \text{const}$. Taking into account the deformation energy, Eqn. (1) can be integrated with respect to the thickness l of the deformable domain from which the cumulative elastic energy is intend to be released, and by multiplying to the area S of this domain, one obtains an equation describing the energy stream accumulation and the relaxation in time in the deformable domain:

$$\frac{dQ}{dt} = -\frac{Q}{t_0} - \lambda_E(t) \quad (2)$$

in which the energy stream on the boundary of the deformable domain and the bulk particles is given by:

$$\lambda_E(t) = \frac{\Delta l}{\mu \eta} \tau(0,t) \quad (3)$$

where $\tau(0,t)$ denotes the total stress on the plane $x = 0$, i.e. on the mentioned boundary. Taking into account that the influence of the shear resistance on the accu-

mulation and relaxation of the elastic energy is negligible, i.e. $\tau_V = 0$, then $\lambda_E(t)$ will iterate the shape of the elastic stress through the deformable domain. For an energy stream which is released from a deformable domain, one can write:

$$U = \frac{\delta_E Q}{\gamma_E t_0} \quad (4)$$

in which γ_E denotes the carry coefficient of the energy, which can be expressed as:

$$\gamma_E = \frac{\tau_E}{\tau_E + \tau_V} \quad (5)$$

where τ_E and τ_V are the elastic and viscous stresses, respectively. The physical meaning of the coefficient δ_E will be explained below.

Substitution of expression (4) into Eqn. (2) implies that:

$$\frac{dU}{dt} = -\frac{\Delta l \delta_E}{\gamma_E \mu \eta t_0} \tau_E(0,t) - \frac{U}{t_0} \quad (6)$$

where $\tau_E(0,t)$ is the stress of the elastic deformation at $x = 0$. For elastic energy accumulation in the deformable domain, the inequality $\tau_E < 0$ in Eqn. (6) occurs.

If from Eqn. (6):

$$-\frac{\tau_E(0,t)}{\gamma_E} = \tau(0,t).$$

Then, Eqn. (6) can be rewritten as:

$$\frac{dU}{dt} = \frac{\Delta l \delta_E}{\mu \eta t_0} \tau - \frac{U}{t_0} \quad (7)$$

furthermore Eqn. (7) can be written as:

$$\frac{dU}{dt} = \frac{\delta_E}{c t_0} \tau - \frac{U}{t_0} \quad (8)$$

where $c = \mu \eta / \Delta l$. Equation (8), for a certain dependence of $\tau(t)$ and according to the initial conditions, describes the variation of the releasing energy stream time, for example during an earthquake. In the stationary case, $U(t)$ satisfies the condition:

$$\frac{dU}{dt} = 0 \quad (9)$$

which implies that:

$$U_{st.} = \frac{1}{c} \delta_E \tau \quad (10)$$

The physical meaning of the coefficient δ_E can be seen from Eqn. (10): i.e. it is the ratio of the releasing energy stream to the total stress.

Let the energy stream rise and damping be considered. At first, let there be no loading action applied to the to be investigated domain of rocks. Then assume a constant loading action (i.e. $F = 0$, therefore $\tau = \text{const}$) at instant $\tau = 0$. For Eqn. (8), the initial condition is given as:

$$U(0) = 0 \quad (11)$$

Then the solution for Eqn. (8) can be written as:

$$U = U_{st.} \left[1 - \exp\left(-\frac{t}{t_0}\right) \right] \quad (12)$$

After the loading action is taken off (i.e. $\tau = 0$), the energy stream damping corresponds to the initial condition:

$$U(0) = U_0 \quad (13)$$

According to Eqn. (13), from Eqn. (8) one obtains:

$$U = U_0 \exp\left(-\frac{t}{t_0}\right) \quad (14)$$

A comparison between Eqn. (12) and (14) shows that it is easier to measure t_0 from the relaxation curve of the energy stream.

Having obtained the energy stream relaxation curve, one can make the dependence $\ln U/U_0(t)$. It follows from Eqn. (14) that:

$$\ln \frac{U}{U_0}(t) = -\frac{t}{t_0} \quad (15)$$

i.e. the above dependence has the form of a straight line. From the slope of this straight line, one obtains the viscoelastic deformation time t_0 :

$$t_0 = -\Delta t \left(\Delta \ln \frac{U}{U_0} \right)^{-1} \quad (16)$$

The probability of viscoelastic deformation

According to MELKONYAN (1997), elastic and viscous deformations probabilities are assumed as:

$$R_E = \frac{1}{t_{0E}}; R_V = \frac{1}{t_{0V}} \quad (17)$$

where $t_{0E} = (\eta/\mu)_{\eta \approx \text{const.}}$, $t_{0V} = (\eta/\mu)_{\mu \approx \text{const.}}$ are the elastic and viscous deformations times, respectively. The total probability of the viscoelastic deformation is assumed to be (e.g. MELKONYAN, 1997):

$$R_D = \frac{1}{t_0} = \frac{1}{t_{0E}} + \frac{1}{t_{0V}} = \frac{t_{0E} + t_{0V}}{t_{0E}t_{0V}} \quad (18)$$

Taking into account the deformation probability, the coefficient δ_E by its physical meaning can be presented as the ratio of the elastic deformation probability to the total deformation probability, i.e.:

$$\delta_E = \frac{R_E}{R_D} \quad (19)$$

Combining Eqns. (17), (18) and (19), one obtains:

$$\delta_E = \frac{t_0}{t_{0E}} \quad (20)$$

According to Eqn. (12), for a loading duration $\Delta t \gg t_0$ one obtains:

$$U = U_{st.} \quad (21)$$

Substitution of Eqn. (21) into Eqn. (10) yields:

$$\frac{U}{\tau} \sim \delta_E \quad (22)$$

or, see Eqn. (20):

$$\frac{U}{\tau} \sim \frac{t_0}{t_{0E}} \quad (23)$$

Viscoelastic deformation efficiency

Let the ratio t_0/t_{0E} be denoted by W_1 and called the efficiency of viscoelastic deformation

$$W_1 = \frac{t_0}{t_{0E}} \quad (24)$$

Substituting expression (18) into Eqn. (24), one obtains:

$$W_1 = \frac{t_{0V}}{t_{0E} + t_{0V}} \quad (25)$$

Equation (23) implies that at $\Delta t \gg t_0$, i.e. for a long duration of the loading, the ratio U/τ depends on both the elastic and the viscous deformation times.

At $\Delta t \ll t_0$, from Eqn. (12) one obtains:

$$U(t) = U_{st.} \frac{\Delta t}{t_0} \quad (26)$$

Substitution of Eqn. (10) into Eqn. (26), using Eqn. (20), implies that:

$$\frac{U}{\tau} \sim \frac{\Delta t}{t_{0E}} \quad (27)$$

It follows from Eqn. (27) that the dependence $t_{0E}(\tau)$ can be obtained from the dependence U/τ on τ .

Activation energy of viscoelastic deformation

It is known that in solids as well as in liquids, the viscous flow is a diffusion phenomena. In this case the stress state leads to certain deflections of the direction of the diffusion components and the absolute quantity.

The process is named viscous flow in liquids and pseudoviscous flow in solids (e.g. VERHOOGEN *et al.*, 1974). The velocity of this process depends on the temperature. The process equation contains the Boltzmann's constant k and the activation energy E_a (e.g. VERHOOGEN *et al.*, 1974):

$$\frac{1}{\eta} = A \exp\left(-\frac{E_a}{kT}\right) \quad (28)$$

where A is a constant.

However, according to Eqn. (4), from Eqn. (28), the temperature dependance of the viscoelastic deformation energy stream can be obtained:

$$U \sim \exp\left(-\frac{E_a}{kT}\right) \quad (29)$$

For the case $\Delta t \ll t_0$, according to Eqns. (23) and (29), U depends on the temperature only through the elastic deformation time t_{0E} . Therefore, the activation energy values obtained at $\Delta t \ll t_0$ will be stipulated by the temperature dependence of the elastic deformation time:

$$U \sim \exp\left(-\frac{\Delta E_{aE}}{kT}\right) \quad (30)$$

where ΔE_{aE} is the activation energy of elastic deformation.

The viscoelastic deformation energy stream ratios, obtained for the case of $\Delta t \gg t_0$ and $\Delta t \ll t_0$ for the same value of τ , according to Eqns. (23) and (27), must be equal to the ratio $t_0/\Delta t$. Then, it can be assumed that:

$$\frac{U(\Delta t \gg t_0)}{U(\Delta t \ll t_0)} \sim \frac{t_0}{\Delta t} \quad (31)$$

where $U(\Delta t \gg t_0)$ and $U(\Delta t \ll t_0)$ are the elastic deformation energy streams at an applied force duration of $\Delta t \gg t_0$ and $\Delta t \ll t_0$, respectively.

Taking into account that in rocks the deformation is, in principal, viscous (e.g. VERHOOGEN *et al.*, 1974), i.e. probabilities ratio $\delta_E = R_E/R_D \ll 1$, and therefore $t_0/t_{0E} \ll 1$, see Eqn. (24), then from the temperature dependence of $U(\Delta t \gg t_0)/U(\Delta t \ll t_0)$, the activation energy of viscous deformation, ΔE_{aV} , is obtained i.e.:

$$U \sim \exp\left(-\frac{\Delta E_{aV}}{kT}\right) \quad (32)$$

Analysis of the dependence (29) shows that for a viscoelastic deformation energy stream at $\Delta t \gg t_0$, taking into account Eqn. (23), $U \sim t_0/t_{0E}$, whence follows:

$$U \sim \exp\left(-\frac{\Delta E_a}{kT}\right) = \exp\left(-\frac{\Delta E_{aE} - \Delta E_{aV}}{kT}\right) \quad (33)$$

Thus, for the case of $\Delta t \gg t_0$, the values of the activation energy ΔE_a obtained correspond to the difference of the thermal activation energies of the elastic and viscous deformations energy carriers:

$$\Delta E_a = \Delta E_{aE} - \Delta E_{aV} \quad (34)$$

Conclusions

The above-mentioned model, which considers the simultaneous appearance of both elastic and viscous behaviours of rocks, at the same time allows the elastic and viscous deformation times, as well as their activation energies to be separated. From this model many other characteristics of the viscoelastic deformation of rocks can be obtained. For example, from Eqn. (18), it can be seen that the energy stream relaxation time t_0 may be increased both with respect to an increase of t_{0E} and t_{0V} . Eqn. (24) shows that the viscoelastic deformation efficiency decreases with increasing elastic deformation time and increases with increasing viscous deformation time. From the dependence of the viscoelastic deformation efficiency W_I on t_0 one can determine which deformation time (elastic or viscous) is varied. If W_I increases with t_0 , then $t_{0E} = \text{const.}$, if W_I decreases, then $t_{0V} = \text{const.}$

The value of the activation energy of the viscoelastic deformation ΔE_a in the stationary case is connected with neither the elastic nor the viscous deformation activation energies but is the result of the superposition of these energies, see Eqn. (34).

The above methods for the definition of the characteristic time t_0 and the model allow the characteristics of the viscoelastic deformation to be determined, which could enable the viscoelastic deformation process to be controlled.

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Резиме

Мобилизација енергије при високоеластичним деформацијама стена

Модел који је анализиран у овом раду приказује симултану појаву (у интермолекуларној размери) еластичног и вискозног понашања стена током њиховог вискоеластичног деформисања. На тај начин омогућено је дефинисање времена релаксације t_0 вискоеластичне деформационе енергије. Када се ово време одреди, онда могу да се раздвоје активирани енергије еластичних и вискозних деформација од температурне зависности вискоеластичног деформационог енергетског тока. Пробабилистички приступ, примењен на вискоеластичне деформације стена, омогућава да се уведе појам тоталне вероватноће, која се састоји од вероватноће еластичних и вискозних деформација.

На тај начин предложени модел разматра симултану појаву еластичног и вискозног понашања стена и у исто време омогућава да се раздвоји еластично и вискозно време деформисања као и њихове активирани енергије. Помоћу овог модела могу да се одреде и многе друге карактеристике вискоеластичног деформисања стена. На пример: енергетски ток времена релаксације t_0 јача са повећањем како еластичног тако и вискозног времена деформације; вискоеластична деформациона ефикасност се смањује када се еластично деформационо време повећава, а расте када се вискозно деформационо време повећава. На основу зависности између вискоеластичне деформационе ефикасности и t_0 , може да се одреди које се деформационо време мења (еластично или вискозно). Ако се деформациона ефикасност повећава са t_0 , онда је еластично деформационо време константно, а ако се деформациона ефикасност смањује са t_0 , онда је вискозно деформационо време константно. Величина вискоеластичне деформационе активирани енергије, у стационарном случају, није повезана ни са еластичним а ни са вискозним деформационим активираним енергијама, али је последица суперпозиција ових енергија.